Design and analysis of a diffraction-limited cat's-eye retroreflector

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Abstract. We describe a design for a modified, cat's-eye retroreflector. The design is catadioptric, containing a single concave mirror and several lenses. This retroreflector design exhibits a unique combination of performance characteristics. It is diffraction limited over a large field angle while operating with a large aperture and numerical aperture. There is little vignetting of the optical beam, even at large field angles, providing good light return at all angles of incidence. It brings the light beam to a focus and allows access to the light near the focal plane, thus decoupling the size of the aperture from the size of devices used with the retroreflector. This final feature makes the design appealing for use with spatial light modulators, or other optical or electro-optical components.
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1 Introduction

Retroreflectors are common components in optical systems. By using three plane surfaces, the well-known cube-corner reflector is constructed. An alternative is the cat's-eye retroreflector, which consists of a lens with a primary mirror at the focus of the lens, or of two mirrors, with the secondary mirror at the focus of the primary.[1,2] These two alternatives are illustrated in Fig. 1. The most significant difference between the cube-corner and cat's-eye designs is that the light entering the aperture of the cat's-eye reflector is brought to a focus, typically at a reflecting surface, while the light traveling through a cube-corner reflector is not. This difference can be significant in a variety of applications. In high power applications, one may wish to avoid bringing an intense beam of light to a good focus. However, in certain situations the ability to focus the light, and then to be able to access that light at or near the focus, can be highly advantageous.

An example of a system that requires a cat's-eye retroreflector is a modulating retroreflector. The modulating retroreflector is designed so that a fixed source sends out an unmodulated beam of light. The light is received by the modulating retroreflector, a signal is placed on the beam by means of an optical modulator, and then the light is reflected back to the base. Such a system demands a fairly large aperture operating at a large numerical aperture to collect as much incoming light as possible. Employing a cube-corner reflector in this system leads to the unfortunate requirement that the optical modulator be as large as the aperture of the retroreflector. Perhaps even more demanding is the associated result that the light modulator must be uniform across the entire beam of light, since all of the light passing through the modulator at a given instant must be modulated identically. This leads to challenges in the fabrication of the light modulators. It also leads to constraints on the speed at which the system can send data, as the entire surface area of the modulator acts on the entire beam of light. A larger surface area for the light modulator leads to a lower data transmission rate, since the modulator is RC limited and the capacitance goes as the area.

The cat's-eye retroreflector addresses these difficulties. First, the spatial light modulator as a whole can be smaller than the aperture of the retroreflector if it is placed advantageously within the optical system. The overall size of the modulator is then dictated primarily by the chief ray of the system, that is, by the desired field of view, while the demand on the overall size of the modulator due to the axial ray of the system is quite small. Second, only a small portion of the light modulator will be used at any given time if it is placed near the focal surface in the retroreflector. With the proposed design, the light entering the retroreflector at a given instant will come to a focus near the modulator, so the area of the modulator illuminated at a given time can be quite small, on the order of a square millimeter. The spot of light will, of course, move across the modulator as the location of the source moves relative to the reflector, but only a small area of the modulator will be used at any given time. By designing the modulator array to act as a photodetector array as well, the driving circuitry can determine which region, or pixel, to address. This greatly relaxes requirements on the uniformity of the modulator, as the modulator can be broken into pixels, each of which is uni-

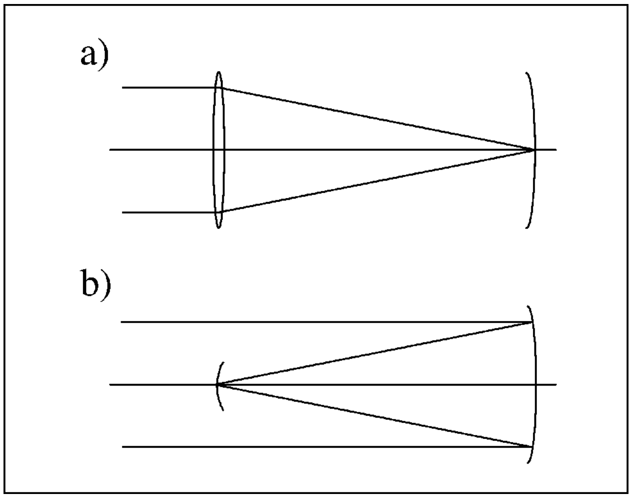

Fig. 1 Basic cat's-eye retroreflector designs: (a) catadioptric system with distance from objective lens to mirror equal to the focal length of the lens, and (b) all-mirror system with the secondary mirror at the focal plane of the objective mirror.

form and can act independently of neighboring regions. This ability to break up, or pixelate, the surface of the modulator also allows for much higher data transmission rates. Clearly, a cat's-eye retroreflector exhibits the performance criteria necessary in this application.

2 Background and System Requirements

The literature related to cat's-eye retroreflectors is not extensive. Beer and Marjaniemi presented an excellent study of wavefront error and construction tolerances for cat's-eye retroreflectors in 1966.[1] In succeeding years, the use of cat's-eye retroreflectors in Fourier transform spectrometers received some attention.[3,4] In the mid- to late-1970s, additional analyses of cat's-eye reflecting systems appeared, extending and building on the work of Beer and Marjaniemi.[5–7] More recently, cat's-eye reflectors that are solid glass have attracted significant attention. These designs are based on a solid sphere of glass of a high refractive index,[8,9] or on hemispheres and spherical shells of glass cemented together.[10] These designs have virtually unlimited fields of view and are quite simple and stable. However, they do not display diffraction-limited performance and do not allow access to the near-focus region of the retroreflector for the optimal placement of a light modulator or similar component. It became apparent that a new design was necessary to meet the needs of the authors.

The requirements on the performance of the desired cat's-eye retroreflector are summarized as follows. The system must bring all incident light to a focus before retroreflecting the light, and allow components to be placed near the focal surface. Hence, the air space in front of the mirror must be large enough to allow the placement of the modulator, which is essentially a plane-parallel plate approximately 0.6 mm thick. The system must be diffraction limited over the full field of view. A fairly large field angle is desirable, with the full field of view preferably being at least 30 deg. This large field of view ensures that the unmodulated beam of light from the fixed source will still be received and returned by the retroreflector as the position of the fixed base changes relative to the retroreflector. The clear aperture of the primary should be about 25 mm in diameter, and the numerical aperture for the system should be 0.25 or greater, giving an infinity f-number of no more than f/2. The size of the primary is dictated by the need to balance two demands. First, the retroreflector must collect from, and return to, the fixed base as much light as possible. Second, the retroreflector must have a relatively small overall package size and weight. The large numerical aperture is motivated by the need to collect as much incident light as possible. As will be seen, the large numerical aperture also limits the required overall diameter of the modulator to less than 15 mm, which is the maximum size of the modulators available for this system. Finally, the intent of the retroreflector is to give a uniformly strong return across the entire field of view of the device. Care must therefore be taken to eliminate any decrease in the effective numerical aperture at large field angles due to vignetting within the system. It should be noted that the retroreflector is to be used with a monochromatic light source, operating in the near infrared at 980 nm. The fact that the retroreflector does not need to be color corrected makes the catadioptric approach reasonable for the system.

The work of Beer and Marjaniemi[1] provided important information for the authors regarding the performance of cat's-eye reflectors. They determined that systems employing a parabolic primary and a spherical secondary provided significantly better performance than systems consisting of a primary and a secondary that were both spherical. The advantage of the parabolic systems was not limited to the on-axis case, but was obvious at all fields of view. While the work of Beer and Marjaniemi was on systems of two mirrors, we used their results as a starting point. In our catadioptric system, we made use of one parabolic surface in our primary, which is a system of lens elements, and used a spherical mirror as our secondary.

The design of this cat's eye retroreflector was executed using a ZEMAX optical design program. The illustrations of the optical system and much of the analysis of this system were prepared using ZEMAX. However, in describing the design of this lens, we attempt to use descriptions general enough so that a specific knowledge of ZEMAX is not required for a full understanding of our results.

3 System Design

As stated earlier, a catadioptric overall system design for the cat's-eye retroreflector was appealing in view of the absence of the need for color correction. The basic system is illustrated in Fig. 2. The objective consists of four refracting elements, with the aperture stop between the second and third elements. The secondary mirror acts as the field stop in the system. A modulator inserted just in front of the secondary mirror would likely serve as the field stop when present in the system. In designing the retroreflector, the system was treated essentially as a telescope. That is, the retroreflector as a whole is an afocal system that will work exclusively at infinite conjugates.

A number of characteristics were incorporated into the design to minimize vignetting at all field angles. While a plane mirror would be desirable for use with the planar modulator and for ease of manufacture and alignment, system designs incorporating a plane mirror suffer from ex-

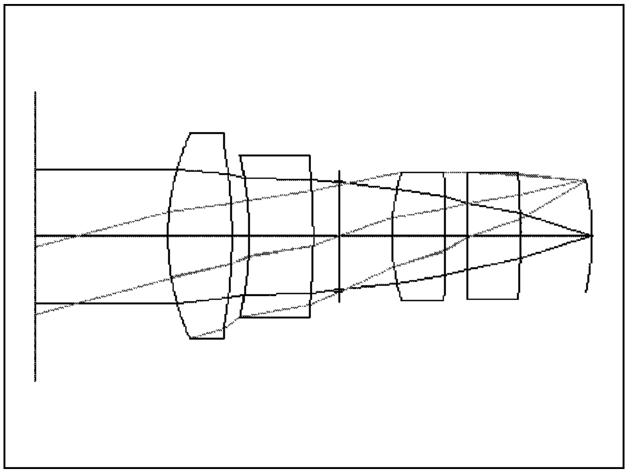

Fig. 2 Basic layout of the cat's-eye retroreflector design. The first surface of the first element is parabolic. All other surfaces are spherical. The aperture stop is between the second and third elements and the mirror is the field stop. The ray bundles passing through the system are for source points on axis and at a field angle of 15 deg. Notice that the extreme rim ray of the off-axis bundle is vignetted by the aperture stop as the light leaves the system.

treme vignetting at larger field angles, significantly diminishing the light returned by the retroreflector. As one would expect, the surface primarily responsible for vignetting is the aperture stop. We overcame the vignetting problem by using a concave secondary mirror. The concave mirror forces the ray bundle to be more symmetric about the chief ray at all field angles, limiting the vignetting losses. The vignetting losses are most effectively limited if the distance from the aperture stop to the mirror is approximately equal to the radius of curvature of the mirror.

Rather than force a fixed separation between the aperture stop and the mirror given a radius of curvature for the mirror, or vice versa, we allowed the optimization routine in ZEMAX to choose the distance and the radius of curvature, within constraints. We constrained the chief ray to have the same angle incoming and outgoing, and to have zero height at the aperture stop as it left the system after reflection. Thus, the retroreflection of the chief ray forced the system as a whole into the desired configuration. In all the designs that performed well, the radius of curvature of the mirror is approximately equal to the aperture stop to mirror separation distance. For example, in the design described in detail later, the ratio of the radius of curvature of the mirror to the separation between the aperture stop and the secondary mirror is 1.01.

Our choice for the radius of curvature of the secondary mirror was driven by the desire for the system to work at large field angles, that is, at least 30-deg full field. Beer and Marjaniemi[1] found a much different optimal solution by modeling a two mirror system with the secondary mirror located at the focal plane of the primary mirror. In their model, the primary mirror is the aperture stop and the secondary is the field stop. They determined that the radius of curvature for the secondary mirror, which minimizes wavefront error when using a parabolic primary mirror and a spherical secondary mirror, is 2.02F, where F is the focal length of the primary mirror, and the distance from the aperture stop to the secondary mirror. While this solution provides good performance at relatively large field angles, up to 10-deg full field, their model did not account for vignetting losses, rendering this approach inappropriate for our system. Snyder[2] carried out a simple analysis of a cat's-eye system and found that the field of view is maximized when the radius of curvature of the secondary mirror is equal to the focal length of the primary lens. For the case of the thin lens objective that Snyder studied, this condition is equivalent to requiring that the radius of curvature of the secondary be equal to the distance from the aperture stop to the secondary mirror. The result of Snyder's analysis is, then, consistent with the results of our design efforts to provide high performance at large fields of view.

It should be mentioned that the feasibility of a telecentric design for the objective was considered, since it offers advantages for a cat's-eye retroreflector. A telecentric objective allows one to achieve retroreflection with a plane secondary mirror, and a plane mirror has the strengths discussed earlier. With the exception of small field angles, however, vignetting losses more than offset this advantage. An outgoing rim ray does not follow the incoming path of the other rim ray, and significant light is vignetted, particularly at the aperture stop. In trial systems tested, only 30 to 50% of the light incident at a field angle of 5 deg was returned by a retroreflector using a telecentric objective.

The utility of a concave secondary mirror, centered on the aperture stop of the objective, becomes clear in the context of a telecentric system. The advantage of a telecentric objective and a plane secondary mirror is that the chief ray always strikes the mirror normally. A concave mirror centered on the aperture stop also has this advantage. However, the concave mirror also allows the rim rays to be reflected symmetrically. The angles of incidence of the upper and lower rim rays are approximately equal at all field angles, not just on axis. The outgoing rim ray therefore follows the incoming path of the other rim ray and virtually no light is vignetted. In the system described later, over 99.8% of the light entering the entrance pupil is returned by the retroreflector. Given the design process followed, we refer to the system with a concave mirror centered on the aperture stop of the objective as ''locally symmetric'' or ''locally telecentric,'' since it incorporated some of the advantages of a telecentric system while limiting vignetting.

Finally, root mean square wavefront error was chosen as the figure of merit for system performance. This is appropriate for a retroreflector. A retroreflector should return a wavefront identical to the initial wavefront along the angle of incidence. In an afocal system such as this, wavefront error is a better indicator of performance than spot size.

4 Description of the Design and Performance Criteria

By employing the basic criteria described before to the design of a cat's-eye retroreflector, several excellent designs were developed. One of them is illustrated in Fig. 2. The ray bundles for an on-axis source and for a source at the maximum, half-field of view of 15 deg are illustrated on the system. Note that an extreme rim ray from the off-axis source is vignetted by the aperture stop as the light leaves the optical system. The retroreflector was optimized for operation at 980 nm. The radius, thickness, and glass information is provided in Table 1. The radius of the aperture

Table 1 Prescription data for the optimal system design. Surface 1 is parabolic. All other surfaces are spherical. Note that the thickness information refers to the thickness after the surface of the same number.

Surface number	Radius of curvature, mm	Thickness after surface, mm	Glass type
1	29.724	7.786	Schott KF9
2	−63.404	2.133	Air
3	−41.517	7.786	Hoya TAFD30
4	−86.211	3.101	Air
5, stop	Infinity	6.489	Air
6	27.693	6.489	Schott SF18
7	−169.05	2.656	Air
8	389.31	6.489	Schott KZFSN2
9	−108.37	8.687	Air
10	−31.061	—	—

stop, surface 5, is 6.730 mm to yield an entrance pupil diameter of 16.2 mm. All surfaces are spheres except for the first surface, which is parabolic.

The focal length of the primary lens system is 40.4 mm. The primary has a numerical aperture of 0.30 and an infinity f/# of 1.70. These values for the numerical aperture and the infinity f/# refer to the ray bundle that the primary focuses onto the secondary mirror. The retroreflector as a whole is, of course, afocal. The large numerical aperture is desirable given the need to collect as much incident light as possible. The large numerical aperture is also key to limiting the diameter of the modulator to 13.8 mm by keeping the secondary mirror close to the aperture stop. The farther the secondary mirror is from the aperture stop, the larger the modulator will have to be for a given field of view. While the system described here has a full field of view of 30 deg, designs for cat's-eye retroreflectors with a full field of view up to 36 deg have been developed as well. The design incorporates elements of reasonable thicknesses and radii of curvature. The design also provides ample space in front of the secondary mirror to insert a spatial light modulator or similar device.

The overall system performance is excellent. Plots of wavefront error are provided at three field angles, on axis, 8 and 15 deg, in Fig. 3. It is worth emphasizing that this wavefront error is the wavefront error of the final wavefront exiting the system. Hence, the exiting wavefront is very similar to the planar wavefront incident on the system. The similarity of the incident and exiting wavefronts is the single most important figure of merit for a retroreflecting system. The wavefront error is largest at about 8 deg with a peak-to-peak wavefront error of 0.12λ. This is well within the nominal diffraction limit of 0.25λ, which is beneficial in tolerancing and the manufacturing of the optical system. The low wavefront error provides excellent light return and is especially notable when combined with the near absence of vignetting in the system. The system performance is enhanced by the flexibility of using high index glasses without concern for dispersion.

Tolerancing was carried out on the system by means of an inverse sensitivity analysis. It was found that diffraction-limited performance can be expected from systems that are manufactured using reasonable tolerances, and this optical system is currently being manufactured. Several parameters of the system were particularly sensitive to manufacturing errors leading to the degradation of system performance. The worst offenders as determined in the tolerancing pro-

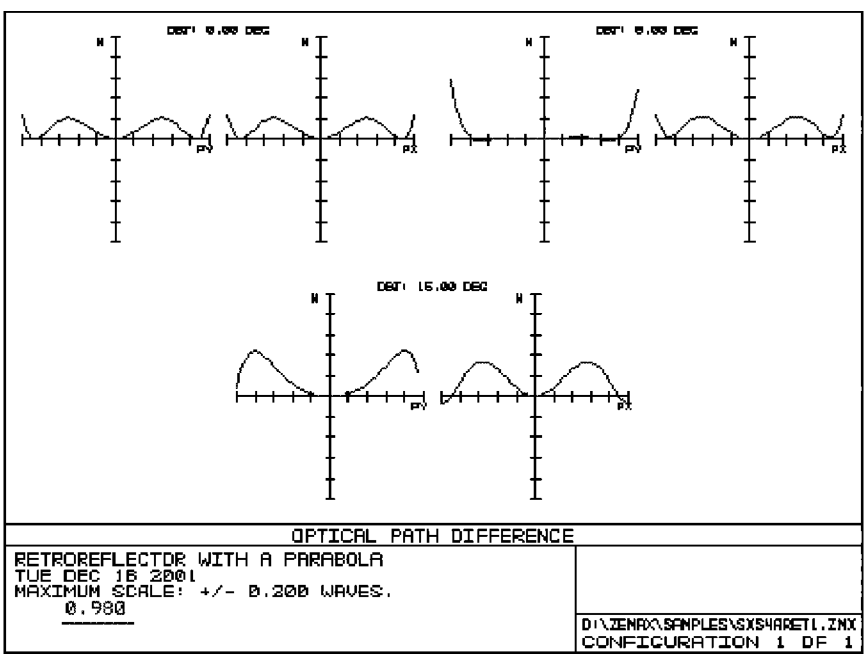

Fig. 3 Wavefront error as a function of aperture at three field angles for the optimal design. In this design the first surface is parabolic. The aperture coordinates are relative and the scale for the optical path difference runs from 0.2 to −0.2 waves. The field angles are 0, 8, and 15 deg. While rms wavefront error is a maximum at 15 deg, the maximum peak to valley error is at 8 deg.

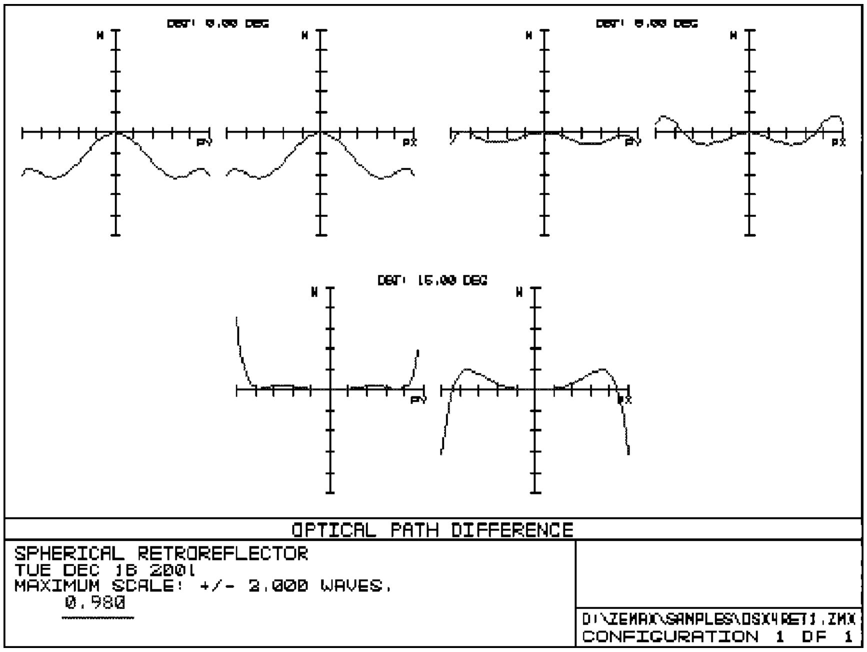

Fig. 4 Wavefront error as a function of aperture at three field angles for the design using only spherical surfaces. The aperture coordinates are relative and the scale for the optical path difference runs from 2.0 to -2.0 waves, an order of magnitude larger than the scale for the optimal design. The field angles are 0, 8, and 15 deg. The rms wavefront error and the peak to valley wavefront error both reach maxima at 15 deg.

cedure include the radius of curvature of the first surface of the system, the parabolic surface, the thickness of the first element, and the distance from the fourth lens to the secondary mirror. It was also determined that the second lens element and the secondary mirror are particularly sensitive to tilt errors. The second and third elements and the secondary mirror are sensitive to decentering errors.

It is worthwhile to note the performance of the retroreflector with respect to various aberrations. It was absolutely essential to minimize distortion in this design as the wavefront tilt caused by distortion rapidly degrades the effectiveness of the system as a retroreflector. That is, a nice flat wavefront is not of much use unless it is going along the same line as the incident light. The distortion is very small in this system, never exceeding 0.001%. Petzval field curvature is naturally limited in a system such as this. This result follows because the system of interest is essentially a telescope with no magnifying power, and Petzval field curvature can be minimized by minimizing the power of a telescope.[11] This circumstance is one of the few situations in which the inherent performance demands for a retroreflector coincide with conditions that optimize performance. The fact that this is a double-pass system also aids in controlling distortion and coma. While this double-pass system is not symmetric about a central stop, it is symmetric about the secondary mirror, which does not eliminate distortion and coma, but does limit it. Astigmatism is the limiting aberration with respect to field of view. The use of the parabolic surface helps to limit the spherical aberration and coma. However, spherical aberration is still the limiting aberration as the numerical aperture increases in size. Astigmatism and spherical aberration are the largest contributing aberrations, consistent with the roles they play in limiting field of view and aperture, respectively. Their contributions are followed by that of Petzval field curvature, with much smaller contributions arising from coma and distortion.

The importance of the single parabolic surface to this design is striking. By making use of all the techniques described previously, a design was completed that is essentially identical to our optimal result, with one exception. In the comparison approach, all surfaces were kept as spheres and the system was then optimized. The wavefront error as a function of relative aperture for the on axis, 8- and 15-deg field angle cases for this all-spherical design is illustrated in Fig. 4. While the all-spherical result is only about five times worse than the system containing the parabolic surface at a field angle of 8 deg, the wavefront error is 18 times worse on axis and 27 times worse at the maximum field angle of 15 deg. Given the requirement of diffraction-limited performance, the parabolic surface is a key characteristic of this design.

We should note that our choice of a parabolic asphere was dictated by the end user of the system who wanted only a single parabolic asphere in the system. This requirement was based primarily on manufacturing considerations. While our final system was therefore constrained to contain a parabolic asphere, we also explored the possible utility of other aspheres in the design. We completed optimizations of systems in which the conic constant of the asphere was allowed to vary. We found that systems containing elliptical aspheres, with conic constants between -0.41 and -0.46, provided the best overall system performance. The systems containing elliptical aspheres maintained all desired performance criteria, such as limited vignetting, a large field of view, and a large numerical aperture, while reducing the maximum peak-to-peak wavefront error to about 0.06λ. This is only about half the maximum peak-to-peak wavefront error of the system with a parabolic asphere. Given a situation in which an elliptical asphere is acceptable, it would therefore be advantageous to replace the parabola with an ellipse.

The possibility of placing the asphere on a different surface was also considered. The asphere was shifted to all surfaces in the primary, and optimizations of the alternative designs were then performed. In all cases, the system with the asphere on the first surface performed better than systems with the asphere placed on a different surface. Qualitatively this is reasonable, given the large angles of incidence incoming rays make on the first physical surface, and given that the first surface has the largest clear aperture of any surface in the system.

5 Summary

We have presented a design for a cat's-eye retroreflector that meets a number of important performance criteria for a modulating retroreflector system. Guided by previous work, this design is a significant contribution to the development of cat's-eye retroreflectors, providing diffraction-limited performance over a wide field of view at a large numerical aperture. Most importantly for us, it brings the retroreflected light to a focus within the retroreflecting system and allows access to the light near the focal surface. The ability to access the light near the focus within the retroreflector is essential to permit the use of a fast, pixelated spatial light modulator. This allows us to decouple the size of the modulator from the aperture size of the retroreflector in our modulating retroreflector system. We believe this design, and similar designs that encompass many of the same principles, can be of great utility in a variety of optical and electro-optic systems.

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